Mathematics Competition
Indiana University of Pennsylvania
2015

DIRECTIONS:

1. Please listen to the directions on how to complete the information needed on the answer sheet.

2. Indicate the most correct answer to each question on the answer sheet provided by blackening the 'bubble' which corresponds to the answer that you wish to select. Make your mark in such a way as to completely fill the space with a heavy black line. If you wish to change the answer, erase your first mark completely since more than one response to a problem will be counted wrong. Make no stray marks on the answer sheet as they may count against you.

3. If you are unable to solve a problem, leave the corresponding answer space blank on the answer sheet. You may return to it if you have time.

4. Avoid wild guessing since you are penalized for incorrect answers. If, however, you are able to eliminate one or more answers as being incorrect, the probability of guessing the correct answer is correspondingly increased. One-fourth of the number of wrong answers will be subtracted from the number of right answers. Therefore, guessing is discouraged. Due to the length of the test, you are not necessarily expected to finish it.

5. Use of pencil, eraser, and scratch paper only are permitted.

6. You will have 110 minutes of working time to do the 50 problems in the test. When time is called, put down your pencil and wait for additional instructions.

Do not turn this page until directed by the proctor to do so.

Answer Key

1. A	18. E	35. E
2. B	19. B	36. B
3. E	20. B	37. D
4. C	21. D	38. C
5. D	22. C	39. D
6. A	23. B	40. B
7. D	24. C	41. B
8. B	25. A	42. B
9. A	26. D	43. D
10. E	27. C	44. A
11. E	28. B	45. A
12. D	29. D	46. A
13. E	30. C	47. C
14. B	31. C	48. C
15. C	32. B	49. A
16. A	33. C	50. E
17. E	34. D	

1. The line with the greatest slope is:

   A. $9x - 7y = -4$

   B. $12x - 17y = 1$

   C. $6x + 2y = 4$

   D. $2x - 3y = -3$

   E. $y = x$

2. A bag of rabbit food will feed 24 rabbits for 72 days. The number of days it will feed 16 rabbits is:

   A. 48

   B. 108

   C. 96

   D. 64

   E. 144

3. Consider the following system of equations: $y = 13x - 1$ and $y = 10x + b$. When $(x, 1000)$ is a solution to the system, the value of b is:

   A. 0

   B. $3x - 1$

   C. 77

   D. $23x - 1$

   E. 230

4. If $2^{x+1} = 50$, then:

   A. $x = 4$

   B. $5 < x < 6$

   C. $4 < x < 5$

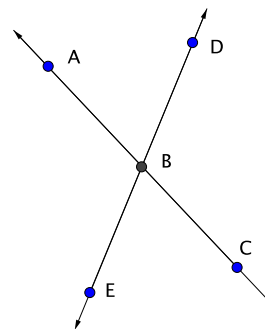   **D.** $x < 4$

   E. None of these

5. Given that $\angle ABD = (9x - 53)°$ and $\angle EBC = (7x - 27)°$, then the values for $\angle ABD$ and $\angle EBC$ are:

    A. $\angle ABD = 13°$ and $\angle EBC = 13°$

    B. $\angle ABD = 128°$ and $\angle EBC = 64°$

    C. $\angle ABD = 13°$ and $\angle EBC = 167°$

    D. $\angle ABD = 64°$ and $\angle EBC = 64°$

    E. $\angle ABD = 83°$ and $\angle EBC = 83°$

6. The only true statement of the following is:
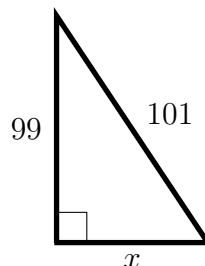
    A. $\sin^{-1}\left(\cos\left(\dfrac{\pi}{3}\right)\right) = \dfrac{\pi}{6}$

    B. $\tan\left(\dfrac{3\pi}{2}\right) = 0$

    C. $\cos(\sin(\pi)) = -1$

    D. $\tan\left(\dfrac{\sqrt{2}}{2}\right) = \dfrac{\pi}{4}$

    E. $\cos^{-1}\left(\sin\left(\dfrac{\pi}{3}\right)\right) = -1$

7. Let (x, y) be the solution to the system of equations $7x - 7y = 14$ and $2x = y + 5$. Then, $-3x - 4y$ is equal to:

    A. 13

    B. 5

    C. -5

    D. -13

    E. -15

8. In the triangle below, the value of x is:

    A. 2

    B. 20

    C. 40

    D. 60

    E. 99

9. A poll was taken, and it was found that 12 students went to the movies on Friday, 10 students went to the movies on Thursday, 4 went on both days, and 7 did not go on either day. The percentage of students who went to the movies only on Thursday is:

    A. 24%

    B. 31%

    C. 30%

    D. 8%

    E. 50%

10. When x and y are nonzero, the expression $\dfrac{\left(\frac{y^2}{x}\right) - \left(\frac{x^2}{y}\right)}{\left(\frac{1}{y}\right) - \left(\frac{1}{x}\right)}$ is equal to:

    A. $x^2 + (xy)^2 + y^2$

    B. $-x^2 + y^2$

    C. $-x^2 + xy - y^2$

    D. $x^2 - xy + y^2$

    E. $-x^2 - xy - y^2$

11. The square of $(\log_{49} 7 - \log_8 64)$ is:

    A. -1

    B. $\dfrac{3}{2}$

    C. $-\dfrac{3}{2}$

    D. $-\dfrac{9}{4}$

    E. $\dfrac{9}{4}$

12. Vaughn, Ashley, and Steve went to the movies with a group of friends and each bought snacks. Vaughn bought two slices of pizza and one bag of popcorn. Steve bought one slice of pizza. Ashley bought one slice of pizza and two bags of popcorn. Vaughn paid $3 more than Steve for his food. Ashley paid $4.25 for her food. The price of one slice of pizza is:

    A. $1.00

    B. $1.25

    C. $1.50

    D. $1.75

    E. $2.00

13. Suppose a circle is inscribed within a square with the radius equal to two. The circle touches the square at the midpoint of each side. Then, the combined area of the shaded regions is:

    A. $16 + 4\pi$

    B. $4 - 4\pi$

    C. 16π

    D. 12

    E. $16 - 4\pi$
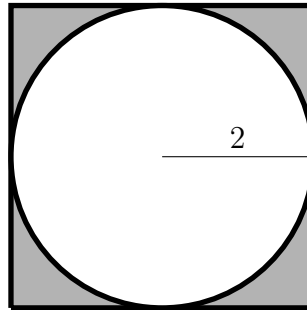
14. In a class there are 15 girls and 10 boys. Three students are randomly selected. The probability that 1 boy and 2 girls are selected is:

    A. $626/118$

    B. $21/46$

    C. $3/25$

    D. $1/48$

    E. $25/56$

15. Of expressions A, B, C, and D given below, the expression that has a numerical value different from the others is:

    A. $4 + 2\sqrt{3}$

    B. $\dfrac{8 + \sqrt{48}}{2}$

    C. $6 + \sqrt{48} - 2 + \sqrt{12}$

    D. $\sqrt{2}(\sqrt{8} + \sqrt{6})$

    E. None of these, they are all equal

16. Given that $f(x) = \log_2(x)$, $g(x) = 8^x$, and $(f \circ g)(a) = 6$, the value of a is:

    A. 2

    B. 3

    C. $\dfrac{1}{3}$

    D. $\sqrt[3]{6}$

    E. None of these

17. A Sportman's Club usually charges a $50 annual membership fee. Next year, the price will be increased to $60. Assume the Club can lose 4 members next year and still collect the same revenue as this year. Then the number of members in the club this year is:

    A. 20 people

    B. 26 people

    C. 27 people

    D. 28 people

    E. None of these

18. The area of a circle is 36π square units. The center of the circle is at $(-2, 3)$. The circle does not pass through the point:

    A. $(-2, -3)$

    B. $(4, 3)$

    C. $(-2, 9)$

    D. $(1, 3 + 3\sqrt{3})$

    E. The circle passes through all of these points.

19. The expression $x+7$ divides evenly into the cubic polynomial $x^3+ax^2-61x+14$. The number of possible values of the variable a is:

    A. 0

    B. 1

    C. 2

    D. 3

    E. Cannot be determined from the given information

20. The solution set, in interval notation, for $\dfrac{(x-1)^3(x-3)}{x(x^2-11x+24)} \geq 0$ is:

    A. $(-\infty, 0) \cup [1, 8)$

    B. $(0, 1] \cup (8, \infty)$

    C. $(0, 1] \cup [3, 8)$

    D. $[1, 3) \cup (3, 8)$

    E. None of these

21. The expression equivalent to $\sec(x)$ is:

    A. $\cos(x) + \dfrac{\tan(x)}{\cos(x)}$

    B. $\dfrac{\sin(x)}{\sec(x)} + \tan(x)$

    C. $\tan(x) - \cos(x)\sin(x)$

    D. $\tan(x)\sin(x) + \cos(x)$

    E. $\dfrac{1}{\tan(x)\csc(x)} - \dfrac{\cos(x)}{\tan(x)}$

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22. Assume $x \neq 0$. If $x + y = 2xy$, then $\dfrac{y^2}{x^2}$ is equal to:

    A. $4x^2 - 4x + 1$

    B. $4x^2 + 4x + 1$

    C. $4y^2 - 4y + 1$

    D. $4y^2 + 4y + 1$

    E. $4y^2 - 4xy + x^2$

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23. In a given sequence, the first term is 3 and each term after that is one less than three times the previous term. The sixth term of the sequence is:

    A. 611

    B. 608

    C. 609

    D. 607

    E. 617

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24. The product of two negative real numbers is 25 and the sum of the two numbers is S. Then, S must satisfy:

    A. $S = -10$

    B. $S \geq -15$

    C. $S \leq -10$

    D. $S \geq -10$

    E. None of these

25. The exact value of $\tan\left(\dfrac{5\pi}{8}\right)$ is:

    A. $-\sqrt{2} - 1$

    B. $\sqrt{2} + 1$

    C. $\dfrac{-\sqrt{2} - 1}{2}$

    D. $\dfrac{\sqrt{2} + 1}{2}$

    E. $\dfrac{-\sqrt{2} + 1}{2}$

26. The interval containing all solutions to $\sqrt{4x} + \sqrt{2x} = \sqrt{4x - 12} + \sqrt{2x + 12}$ is:

    A. $(0, 2)$

    B. $(1, 3)$

    C. $(2, 5)$

    D. $(4, 7)$

    E. $(6, 10)$

27. Let $ABCD$ be a quadrilateral with sides $AB = 6$, $BC = 5$, $CD = 3$, and $AD = 4$. Given that $\angle BAD$ and $\angle ADC$ are both right angles, the area of the quadrilateral is:

    A. 24

    B. 12

    C. 18

    D. Not possible to determine

    E. None of these

28. The number of liters of a 9% acid solution that must be mixed with 5 liters of a 3% acid solution and 4 liters of a 6% acid solution to obtain a solution that is 8% acid is:
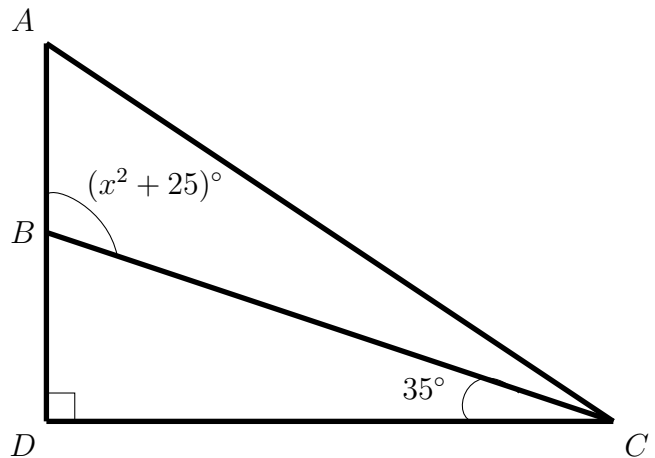
    A. 43

    B. 33

    C. 23

    D. 13

    E. 3

29. If $10a - a^2 = 1$, the value of $a^2 + \dfrac{1}{a^2}$ is:

    A. 20

    B. $40\dfrac{1}{10}$

    C. $38\dfrac{9}{10}$

    D. 98

    E. 102

---

30. In the diagram below, the value of x must be:

    A. $8°$

    B. $9°$

    C. $10°$

    D. $11°$

    E. $12°$

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31. The number of solutions to $4 - |2x + 3| = 2 + |x + 1|$ is:
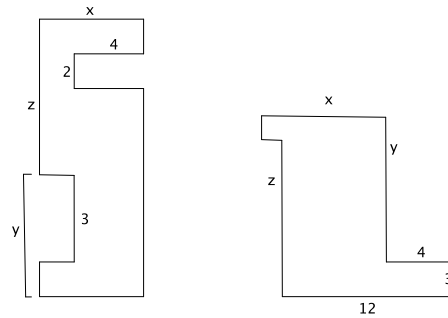
    A. 0

    B. 1

    C. 2

    D. infinite

    E. None of these

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32. The solution set of the equation $(\log_2(x))^2 - \log_2(x^3) = 4$ is:

    A. $\left\{\dfrac{1}{16}\right\}$

    B. $\left\{\dfrac{1}{2}, 16\right\}$

    C. $\{16\}$

    D. $\{1, 16\}$

    E. None of these

33. The total combined area of the two figures below (not to scale) is:

    A. 320

    B. 560

    C. 2xy + 8z + 16

    D. 2xz - 6x - 8z - 8

    E. None of these

34. If four experts and two trainees are working on a project, it takes one day to finish the project. If only two experts are working on a project, then it takes five trainees working with them to finish the project in one day. If one trainee was working alone on a project, the number of days it would take him or her to finish the project is:

    A. 27/6 days

    B. 5 days

    C. 36/7 days

    D. 8 days

    E. 47/9 days

35. A cube has a surface area of 54 square centimeters. The volume of the cube is:

    A. 26 cubic centimeters

    B. 36 cubic centimeters

    C. 54 cubic centimeters

    D. 162 cubic centimeters

    E. None of these

36. Suppose there is a 40% chance of rain on Monday, a 30% chance of rain on Tuesday, and a 50% chance of rain on Wednesday. The probabilty that it will rain on exactly two of the three days is:

    A. 35%

    B. 29%

    C. 26%

    D. 23%

    E. 20%

37. In the triangle below (the figure may not be to scale), the value of x is:

    A. $9\sqrt{3}$

    B. $17\sqrt{3}$

    C. $4\sqrt{13}$

    D. $\sqrt{217}$

    E. $\sqrt{370}$
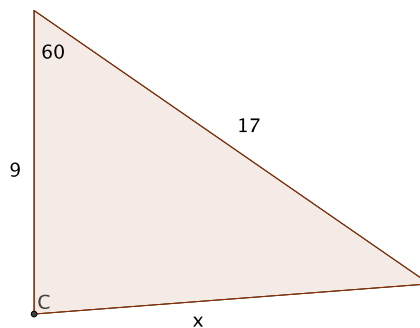
38. A train travels to its destination and back during a trip. If it averages 80 miles per hour on the way and 90 miles per hour on the return, the train's average rate for the entire trip is:

    A. 83 miles per hour

    B. 85 miles per hour

    C. $\frac{1440}{17}$ miles per hour

    D. $\frac{720}{17}$ miles per hour

    E. $\frac{1230}{17}$ miles per hour

39. You have a bag containing 6 red marbles, 4 blue marbles, and 8 purple marbles. In simplest form, the probability of pulling a red marble and not putting it back, then pulling a blue marble and putting it back, and then pulling a purple marble would be:

    A. 16/459

    B. 32/729

    C. 8/153

    D. 32/867

    E. 3/52

40. If $\dfrac{t+3}{t-2} = x$ and $\dfrac{z+1}{z-2} = t$, then z is equal to:

    A. $\dfrac{x+1}{x+4}$

    B. $\dfrac{5x+5}{x+4}$

    C. $\dfrac{x+3}{x-2}$

    D. $\dfrac{x-4}{2x+2}$

    E. None of these

41. A sector of a circle has a radius of 8 inches and an area of 8π square inches. The arc length of the sector is:

   A. π inches

   B. 2π inches

   C. 4π inches

   D. 8π inches

   E. None of these

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42. Two numbers are called a *twin prime pair* when each of the two values are prime and their difference is 2. For example, 149 and 151 form a twin prime pair. The number of twin prime pairs that exist between 0 and 100 is:

   A. 7

   B. 8

   C. 9

   D. 10

   E. None of these

---

43. For a positive real number a, consider

$$2\log_b(x) = 2\log_b(1-a) + 2\log_b(1+a) - \log_b\left(\frac{1}{a} - a\right)^2.$$

   Then x is equal to:

   A. $-a$

   B. $\pm a$

   C. a^2

   D. a

   E. b

---

44. The sum of the distinct real solutions to $x^3 + 21x^2 - x = x^4 + 20$ is:

   A. 1

   B. -1

   C. 3

   D. 11

   E. -11

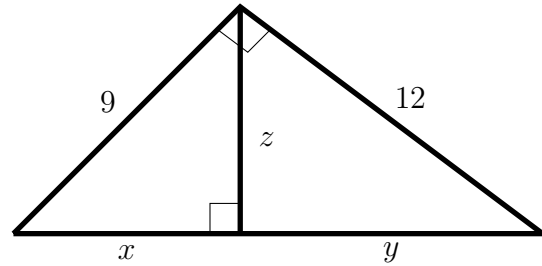45. In the right triangle below, find the values of x, y, and z:

A. $x = \dfrac{27}{5}, \; y = \dfrac{48}{5}, \; z = \dfrac{36}{5}$

B. $x = 27, \; y = 48, \; z = 36$

C. $x = \dfrac{15}{2}, \; y = \dfrac{15}{2}, \; z = \dfrac{3\sqrt{39}}{2}$

D. $x = \dfrac{15}{2}, \; y = \dfrac{15}{2}, \; z = \dfrac{3\sqrt{11}}{2}$

E. None of these

46. Consider the following equation:

$$\frac{14x + 10}{x^4 + 3x^3 - 3x^2 - 7x + 6} = \frac{A}{x + 2} + \frac{B}{x - 1} + \frac{C}{(x - 1)^2} + \frac{D}{x + 3}$$

Then $A^2 + B^2 + C^2 + D^2$ is equal to:

A. 12

B. 16

C. 20

D. 25

E. 36

47. In a cafeteria, there is a circular table seating 10 students. Two seating arrangements are said to be distinct if there exist two students who are neighbors in one arrangement but not in the other arrangement. The number of distinct seating arrangements is:

A. 10!

B. $\dfrac{10 \cdot 9}{2}$

C. $\dfrac{9!}{2}$

D. 9!

E. None of these

48. An interval containing the solution to $\sqrt[3]{x} + \sqrt{3x + 32} = 6$ is:

   A. (-3,-1)

   B. (-1,0)

   C. (0,1)

   D. (1,2)

   E. $(2, \infty)$

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49. A circle is inscribed within a square with a side of 12 units. A regular hexagon is then inscribed within the circle. The hexagon is then divided into six equilateral triangles, and finally, a circle in inscribed within each of these triangles (see the diagram below). The area of the shaded region is:
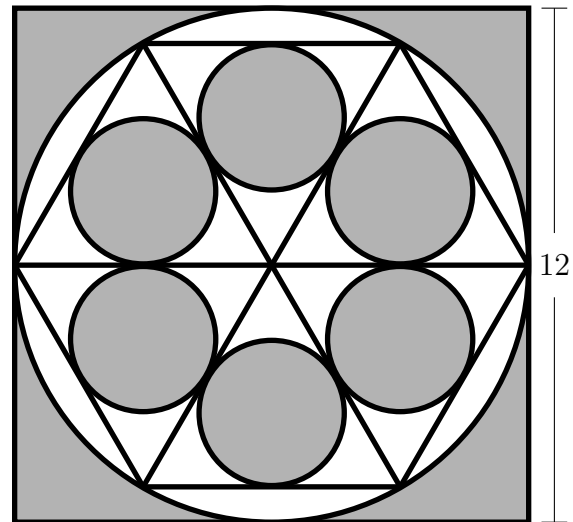
   A. $18(8 - \pi)$ square units

   B. 18π square units

   C. $144 - \frac{63}{2}\pi$ square units

   D. $36(4 - \pi)$ square units

   E. 36π square units

---

50. Given the following diagram (not to scale) the exact area of the larger triangle is:

   A. $\frac{1}{2}\left(89\sqrt{3} + 240\right)$ square units

   B. $\frac{1}{4}\left(139\sqrt{3} + 240\right)$ square units

   C. $\frac{1}{8}\left(89\sqrt{3} + 160\right)$ square units

   D. $\frac{1}{8}\left(139\sqrt{3} + 260\right)$ square units

   E. None of these