

13-161
UWUCC: App-314114
Senate: Info-3/25/14

Undergraduate Distance Education Review Form

(Required for all courses taught by distance education for more than one-third of teaching contact hours.)

Existing and Special Topics Course

Course: MATH 471 Algebra for Elementary and Middle Level Teachers

Instructor(s) of Record: Dr. Edel Reilly

Phone: 7-7907 Email: ereilly@iup.edu

Step Two: Departmental/Dean Approval

Recommendation: Positive (The objectives of this course can be met via distance education)

Negative

[Signature] 2/28/14
Signature of Department Designee Date

Endorsed:

[Signature] 3/3/14
Signature of College Dean Date

Forward form and supporting materials to Liberal Studies Office for consideration by the University-wide Undergraduate Curriculum Committee. Dual-level courses also require review by the University-wide Graduate Committee for graduate-level section.

Step Three: University-wide Undergraduate Curriculum Committee Approval

Recommendation: Positive (The objectives of this course can be met via distance education)

Negative

[Signature] 3/5/14
Signature of Committee Co-Chair Date

Forward form and supporting materials to the Provost within 30 calendar days after received by committee.

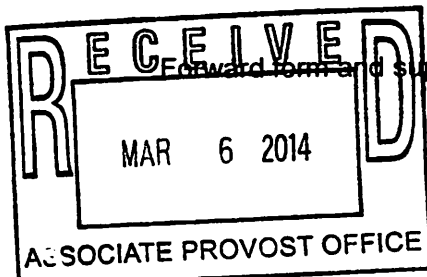
Step Four: Provost Approval

Approved as distance education course

Rejected as distance education course

[Signature] 3/6/14
Signature of Provost Date

Forward form and supporting materials to Associate Provost.



MAR 5 2014

MAR 4 2014

1. How is/are the instructor(s) qualified in the distance education delivery method as well as the discipline?

During my time at IUP I have always used whatever Learning Management System was in place first WebCT, then Moodle, and now D2L. While I have not taught an on-line course before, I do use D2L for students to access course materials and grades. During the summer of 2012 I attended the five workshop “boot camp” IT-Services provided on using dropbox, discussion areas, and quizzes on D2L. I have attended several LMS workshops at IUP and watched many of the videos provided. I participated in three meetings with the university Online Learning Specialist (David Porter) that were held for our Elementary Mathematics Education Committee. These were customized workshops for our planned online classes. I also attended a Reflective Practice Large Group Meeting presented by David Porter on rubrics designed to evaluate the quality of online instruction. Finally, using the Smart Board technology in all of the classrooms I teach, I have recorded several of my lessons and made them available to students on D2L.

2. How will each objective in the course be met using distance education technologies?

Course Objective 1: explain algebra topics taught in the elementary and middle level grades

Course Objective 2: explore real-world situations and problems that involve algebra as a symbolic language useful in many areas of life and as a tool for problem solving.

Course Objective 3: develop and implement a lesson to teach a topic in algebra.

Course Objective 4: demonstrate mastery and experience the different roles algebra plays in the study of patterns, as a symbolic language useful in many areas of life and as a tool for problem solving

Course Objective 5: represent physical situations symbolically, graph linear and quadratic equations and inequalities, and exhibit fluency working with symbols

Course Objective 6: use manipulatives, calculators, and other technologies to develop concepts and solve problems in algebra. Technology is an important topic where explorations and demonstrations are integrated throughout the entire course where appropriate.

Course objectives 1, 2, 4, and 5 have been grouped together as they refer to the algebraic content being studied in this course. The content in the course has been broken down into 7 topic areas. Using a Learning Management System students will view instructor designed presentations. Students will then perform on-line explorations on the content presented. In addition to the explorations there are practice problems and a quiz provided for each of the content areas. Student work will be placed on the LMS and will be evaluated by the instructor. On-line software tools will also be used to evaluate students' work.

Course Objectives 3 and 6: Utilize technology to explore algebra concepts.

Students will explore a variety of technologies for teaching algebra such as the NCTM Illuminations and the National Library of Virtual Manipulatives. In order to demonstrate their use of technology in an algebra classroom, each student will develop and teach an algebra lesson on a topic of their choice that uses technology. In the discussion forum of the LMS, students will share their lesson plan and report on their teaching strategies that use technology.

3. How will instructor-student and student-student, if applicable, interaction take place?

Instructor-student interaction will take place by several methods. The instructor will interact with students via the threaded discussion message board. The instructor will post discussion questions on the message board relating to the assigned readings and inviting students to respond. Instructor-student interaction will also take place via regular email messages to all students regarding general course issues and individual email messages to students. Interaction will also occur during assignment grading when feedback is provided to students. Finally, the instructor will be available as needed for email, chat room, telephone, or face-to-face conversations with individual students during office hours or by appointment.

Student-student interaction will take place in several threaded discussion forums. Students will interact in a discussion forum devoted to the NCTM Illuminations Project and the design of a lesson plan. Both activities will deal with looking at algebraic content and thinking about ways algebra is taught at the elementary and middle level. The students will be asked to share an analysis of various algebraic activities and provide feedback. Students work will be posted on the discussion forum of Learning Management System. Students are then required to comment on 3 or 4 other students' work.

4. How will student achievement be evaluated?

Homework and Quizzes

Students will complete practice problems and on each of the content areas covered in the course. Student work will be placed on the LMS and will be evaluated by the instructor. On-line software tools will also be used to evaluate students' work. The assignments and quizzes account for 30% of the final grade.

Project: *Illuminations Activity*

Students will prepare an evaluative discussion of an algebra lesson at the following electronic resource of the National Council of Teachers of Mathematics: Electronic Resource at NCTM Illuminations: <http://illuminations.nctm.org/>

The students will submit a 3-4 page review/reflection on the lesson that includes the following:

1. Identify and describe specific components of the mathematics content of the lesson.
2. Provide an analysis that includes the essential mathematical ideas students will learn from the activity.
3. Provide a context with specific examples of the ways activity would be used an elementary math class focusing on algebraic thinking or a middle level algebra classroom.
4. Provide **two (2)** adaptations of the lesson so that one can be used as an enrichment activity to be used with students who are accelerated or gifted and **one** can be used for students who may need additional re-enforcement of the concepts.

This will account for 20% of the final grade.

Practice Teaching Project

Students will construct a lesson plan aligned with Common Core Standards for Mathematics. The lesson plan will address an algebraic concept. The plans must incorporate the appropriate use of technology and must address common misconceptions and errors students might have. The lesson plan account for 20% of the final grade.

Assignments and Quizzes	30%
NCTM Illuminations	20%
Lesson Plans	20%
Mid-term and final exam	30%

5. How will academic honesty for tests and assignments be addressed?

Students will be made aware of the IUP Graduate Handbook's Academic Integrity Policy, located at <http://www.iup.edu/page.aspx?id=127235>. In addition to clearly expressing the above expectations in the Graduate Handbook and explicitly referring to these expectations in the syllabus, course evaluative assignments will be designed to minimize the potential for violations of academic integrity. The discussion papers, lesson plans, Illuminations activity, and unit plan will be based on individual experiences from which the students are expected to draw.

Course Syllabus—Online Version

I. Catalog Description

MATH 471 Algebra for Elementary and Middle School Teachers (3c-01-3cr)

Prerequisite: MATH 152

Topics include multiple representations of sequences, integers, expressions, equations, systems of equations, inequalities, and matrices. Representations of expressions and equations will be explored through the use of hands on and visual aides and with appropriate technology. Connections will be made with the teaching and learning of algebraic concepts in elementary and middle schools.

II. Course Objectives

The student will:

1. explain algebra topics taught in the elementary and middle level grades
2. explore real-world situations and problems that involve algebra as a symbolic language useful in many areas of life and as a tool for problem solving.
3. develop and implement a lesson to teach a topic in algebra.
4. demonstrate mastery and experience the different roles algebra plays in the study of patterns, as a symbolic language useful in many areas of life and as a tool for problem solving
5. represent physical situations symbolically, graph linear and quadratic equations and inequalities, and exhibit fluency working with symbols
6. use manipulatives, calculators, and other technologies to develop concepts and solve problems in algebra. Technology is an important topic where explorations and demonstrations are integrated throughout the entire course where appropriate.

III. Course Outline

A. Integer Models

1. Positive and Negative Numbers
2. Operations on Integers

B. Algebraic Representations in middle level grades

1. Linear
2. Two- and Three-Dimensional

C. Sequences

1. Arithmetic, Geometric, Fibonacci
2. Recursive and Explicit Representations

D. Representing Expressions

1. Concretely and Pictorially
2. Abstractly

E. Solving Equations and Inequalities Two Unknowns

1. Graphically
2. Algebraically and by Substitution

- F. Fitting Lines to Data
 - 1. Scatter Plots
 - 2. Median – Median Line
 - 3. Least Squares Regression
 - 4. Calculator fitting
- G. Matrices and Determinants
 - 1. Algebraically
 - 2. With Calculators and Spreadsheets

IV. Evaluation Methods

Distribution of points

Assignments and Quizzes	30%
NCTM Illuminations Activity	20%
Lesson Plan	20%
Mid-term	15%
Final	15%

Assignments and Quizzes

Students will complete assignments including appropriate readings, explorations of algebraic concepts with and without technology, and performance tasks aligned with the Course Outcomes. Student work will be placed on the LMS and will be evaluated by the instructor. On-line software tools will also be used to evaluate students' work. The assignments and quizzes account for 30% of the final grade.

Illuminations Activity

Students will prepare an evaluative discussion of an algebra lesson at the following electronic resource of the National Council of Teachers of Mathematics: Electronic Resource at NCTM Illuminations: <http://illuminations.nctm.org/>

The students will submit a 3-4 page review/reflection on the lesson that includes the following:

1. Identify and describe specific components of the mathematics content of the lesson. Align the lesson to the Common Core State Standards.
2. Provide an analysis that includes the essential mathematical ideas students will learn from the activity.
3. Provide a context with specific examples of the ways activity would be used an elementary math class focusing on algebraic thinking or a middle level algebra classroom.
4. Provide **two (2)** adaptations of the lesson so that one can be used as an enrichment activity to be used with students who are accelerated or gifted and **one** can be used for students who may need additional re-enforcement of the concepts.

This will account for 20% of the final grade.

Lesson Plan

Students will construct a lesson plan aligned with Common Core Standards for Mathematics. The lesson plan will address an algebraic concept. The plans must incorporate the appropriate use of technology and must address common misconceptions and errors students might have. The lesson plan account for 20% of the final grade.

Mid-term and final exams

Using the Quiz feature of the LMS a mid-term and final exam will be administered. These exams will be designed by the instructor and placed on the LMS. Students will have limited time in which to complete both. The mid-term and final exams combined account for 30% of the final grade.

V. Sample Grading Scale

A:	90% - 100%
B:	80% - 89%
C:	70% - 79%
D:	60% - 69%
F:	≤ 59%

VI. Required Textbook

Course packet with readings and materials

VII. Special Resource Requirements

Graphing Calculator

Technical Support

For questions regarding Desire2Learn and using the system, contact the 24/7 Perceptis Helpdesk at 1-877-730-6229 or via the Web at <http://smartipantz.perceptis.com/Indiana>.

To obtain technical support for computer issues related to this course, please contact Indiana University of Pennsylvania's student helpdesk at 724-357-4000, Monday–Friday, between 7:30AM and 5:30PM Eastern Time (ET). You should be prepared to give specific details regarding your technical issue(s), including what you were doing before the error occurred and the exact text of any error messages received. If you experience issues outside of the normal helpdesk hours, you can also submit your error via e-mail at it-support-center@iup.edu or via electronic form available online at <http://www.iup.edu/itsupportcenter/help>.

If you are not familiar with Distance Education, please visit this site for guidance. (Library Services) <http://www.iup.edu/page.aspx?id=77153>

VIII. Bibliography

Cuevas, G.J. (2001). Navigating through algebra, Grades 3-5. Reston, VA: NCTM.

- Coxford, A.F. (1988). The ideas of algebra, K-12, 1988 NCTM Yearbook. Reston, VA: NCTM.
- Driscoll, M. (1999). *Fostering Algebraic Thinking*. Portsmouth, NH: Heinemann.
- Friel, S. et al. (2001). *Navigation through algebra, Grades 6-8*. Reston, VA: NCTM.
- Heid, K. (1995). *Algebra in a technological world*. Reston, VA: NCTM.
- Kaseberg, A. (2007). *Introductory algebra: A just-in-time approach (4th ed.)*. Boston, MA: PWS Publishing Co.
- Lawrence, A., & Hennessy, C. (2002). *Lessons for algebraic thinking, Grades 6-8*. Sausalito, CA: Math Solutions Publications.
- Murdock, J., Kamischke, E., & Kamischke, E. (2002). *Discovering algebra: An investigative approach*. Emeryville, CA: Key Curriculum Press.
- National Council of Teachers of Mathematics. (2000). *Principles and standards for school mathematics*. Reston, VA: NCTM.
- Stein, R. G., & Wallace, L. (2006). *Mathematics for teachers: An exploratory approach to arithmetic, algebra, and geometry*. Reno, NV: Bent Tree Press.
- Stump, S., Roebuck, K., & Bishop, J. (2008). *Algebra for elementary and middle school teachers: An inquiry approach*. New York: Pearson Custom Publishing.
- Wah, A. and Picciotto, H. (1994). *Algebra: themes, concepts, and tools*. Mountain View, CA: Creative Publications.
- Wickett, M. et al. (2002). *Lessons for algebraic thinking, Grades 3-5*. Sausalito, CA: Math Solutions Publications.

Currently Approved Syllabus of Record

I. Catalog Description

MATH 471 Algebra for Elementary/Middle Level Teachers (3c-01-3cr)
 Prerequisite MATH 152

Topics include multiple representations of sequences, integers, expressions, equations, systems of equations, inequalities, and matrices. Representations of expressions and equations will be explored through the use of hands on and visual aides and with appropriate technology. Connections will be made with the teaching and learning of algebraic concepts at the Elementary/Middle Level.

II. Course Outcomes

Students will:

- A. explain algebra topics taught at the Elementary/Middle Level.
PDE Guideline: II.B.2.a
- B. explore real-world situations and problems that involve algebra as a symbolic language useful in many areas of life and as a tool for problem solving.
PDE Guideline: II.B.2.f
- C. develop and implement a lesson to teach a topic in algebra. PDE Guideline: II.B.2.d
- D. master the different roles algebra plays in the study of patterns, as a symbolic language useful in many areas of life and as a tool for problem solving. PDE Guideline: II.B.2.f
- E. represent physical situations symbolically, graph linear and quadratic equations and inequalities, and exhibit fluency working with symbols. PDE Guideline: II.B.2.h
- F. demonstrate appropriate uses of manipulatives, calculators, and other technologies to develop concepts and solve problems in algebra. PDE Guideline: II.B.6.f

Course Outcome	College Conceptual Framework / Danielson	INTASC Standard/ Principle	NCTM—Standards for Middle Level Mathematics Teachers	Course Assessment Measuring Outcome
1	1	1, 4	2.3 9	Key Assessment: Midterm
2	1	1, 7	2.3 1, 4	Projects, Quizzes, Activities Midterm & Final Exam
3	1, 2, 3 4	6-9	2.3, 3.1 - 3.5 16	Project
4	1a	1, 4	2.3 10	Projects, Quizzes, Activities Midterm & Final Exam
5	1a	1, 4	2.3 10	Projects, Quizzes, Activities Midterm & Final Exam
6	1		2.3 6	Projects, Quizzes, Activities Midterm & Final Exam

III. Detailed Course Outline

- A. Integer Models (Outcomes #1, #6) 3 academic hours
 - 1. Positive and Negative Numbers
 - 2. Operations on Integers
- B. Algebraic Representations in middle level grades (Outcome #5) 6 academic hours
 - 1. Linear
 - 2. Two- and Three-Dimensional
- C. Sequences (Outcome #4) 6 academic hours
 - 1. Arithmetic, Geometric, Fibonacci
 - 2. Recursive and Explicit Representations
- D. Representing Expressions (Outcome #2) 3 academic hours
 - 1. Concretely and Pictorially
 - 2. Abstractly
- E. Solving Equations and Inequalities Two Unknowns (Outcome #4) 6 academic hours
 - 1. Graphically
 - 2. Algebraically and by Substitution
- F. Fitting Lines to Data (Outcomes #5, #6) 6 academic hours
 - 1. Scatter Plots
 - 2. Median – Median Line
 - 3. Least Squares Regression
 - 4. Calculator fitting
- G. Matrices and Determinants (Outcomes #5, #6) 3 academic hours
 - 1. Algebraically
 - 2. With Calculators and Spreadsheets
- H. Student Presentations (Outcomes #3) 6 academic hours

This syllabus covers 39 academic hours leaving 3 academic hours for testing and/or review. The final is an additional 2 academic hours.

IV. Evaluation Methods

Distribution of points:

Class work / participation / daily work	30%
Presentations / projects / portfolios / journals	20%
Practice Teaching Project	20%
Midterm (Key Assessment)	15%
Final	15%

The Midterm is the Key Assessment and shall be required of all instructors.

V. Grading Scale

A = 90% - 100%

B = 80% - 89%

C = 70% - 79%

D = 60% - 69%

F < 60%

VI. Attendance Policy

The course attendance policy is consistent with the university policy.

VII. Required Textbook

Course packet with readings and activities

VIII. Special Course Requirements

None.

IX. Bibliography

Cuevas, G.J. (2001) Navigating through algebra, Grades 3-5. Reston, VA: NCTM.

Coxford, A.F. (1988) The ideas of algebra, K-12, 1988 NCTM Yearbook. Reston, VA: NCTM.

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Lawrence, A., & Hennessy, C. (2002) Lessons for algebraic thinking, Grades 6-8. Sausalito, CA: Math Solutions Publications.

National Council of Teachers of Mathematics. (2000) Principles and standards for school mathematics. Reston, VA: NCTM.

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Wah, A. and Picciotto, H. (1994) Algebra: themes, concepts, and tools. Mountain View, CA: Creative Publications.

Wickett, M. et al. (2002) Lessons for algebraic thinking, Grades 3-5. Sausalito, CA: Math Solutions Publications.

Sample Course Module

MATH 471—Algebra for Elementary and Middle School Teachers

Representing Expressions

This unit examines models that can be used in the teaching of expansion of expressions of the format $a(b + c)$, $(ax + by)(cx + dy)$, $(ax + by)^2$.

Emphasis will be placed on connecting abstract ideas to concrete models.

Combining Like Terms—power point slides provided following page 11

1. Discussion Board Activity
 - a. Comment on the two models (perimeter/coins in bags). Is one more appealing than the other?
 - b. Develop another concrete model to model of combining like terms.
 - c. Do the models always work? How would you model $(2a - 2)^2$ $(3x - 2y)^2$?

2. Game: Combining like Terms

Use a square grid placing the designated number of red (r) and yellow (y) counters in each cell. For example:

4r	y	3r	3r	8y	r	2y	End
3y	2r	7y	4y	6r	y	2r	
r	4y	5r	5y	2r	2y	4y	
5y	6r	2y	r	3y	r	4r	

Start

Moving from Start to End from one cell to the other—to the right (R) or up (U)—only a number of possible questions can be looked at:

- (i) Given a route UURRRURRRUR what number of red and yellow counters do you collect following that route?
- (ii) What is the route to be followed if you want the number of red counters to be as small as possible? What is the route to be followed if you want the number of yellow counters to be as small as possible?
- (iii) What is the route to be followed if you want the number of red counters to be as large as possible? What is the route to be followed if you want the number of yellow counters to be as large as possible?
- (iv) Which route will give the least TOTAL of counters? Which route will give the maximum number of counters?
- (v) Change the up/right movement can be changed to up /down/right/left (diagonal). How will items (ii) – (iv) change.

Using the discussion board on D2L share how you would find a game like this useful to help middle level students understand combining like terms.

Combining Like Terms

Warm Up

- Write an algebraic expression
 1. Seven less than x
 2. The product of 7 and y
 3. The quotient of 15 and x
 4. Ten taken from the product of six and n
 5. Name the three main properties of numbers and give an example of each.

Combining Like Terms

Key Vocabulary needed:

- Constant
- Variable
- Coefficient
- Exponent

Prior Knowledge needed

- ✓ Integer review
- ✓ Laws of Exponents
- ✓ Distributive Property

General Overview

TERMS

- Numbers \Rightarrow Constants $\Rightarrow 2, -4, 3/4$
- Variables $\Rightarrow a, x, y$
- Product of a Number and Variable $\Rightarrow 3x, -57z$,
Product of Variables $\Rightarrow mn, xy, ac$
- Variable raised to a power $\Rightarrow v^3, x^2$

LIKE TERMS

- All numbers $\Rightarrow 2$ & 56 are like terms $\Rightarrow 8$ & $6/8$ are like terms.
- Terms with the same variable and the same exponent. $\Rightarrow m^2$ & m^2 are like terms $\Rightarrow 3z$ & $5z$ are like terms. $\Rightarrow x^2y^4$ & $10x^2y^4$ are like terms.

COMBINE

You can add or subtract terms that are alike.

- $2 - 56 = -54$
- $8 + 6/8 = 8.75$
- $m^2 + m^2 = 2m^2$
- $3z - 5z = -2z$
- $x^2y^4 + 10x^2y^4 = 11x^2y^4$

Conceptual Understanding

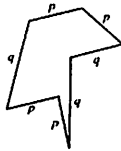
- Avoid seeing 'letters' as labels of objects. That letters are representing variables should be clear in the model.
- What is clearly to be avoided is 'fruit salad and animals' algebra. While $a + a$ can be looked at as one apple and another apple makes 2 apples so $a + a + 2a$, or $3c + 2g + c + 4g$ as 3 cows and one cow make 4 cows, two goats and four goats makes 6 goats, so $3c + 2g + c + 4g = 4c + 6g$. Although this gives 'correct' answers, the conceptual idea (letters representing objects) is invalid (see previous lesson on variables)
- Use concrete models to make connections.

Perimeter model

- You have different rods, some of the same length and others of different lengths. You did not measure them, but you know that some rods have a length of p cm, others of q cm, etc. You have also some rods which you know are 1 cm in length. The rods are used to make polygonal shapes and you find their perimeters in terms of $p, q,$ etc.

Example 1

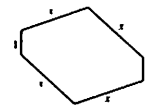
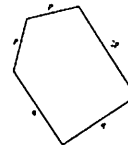
- What is the perimeter of this polygon? The lengths of the sides are in cm.



- Give the perimeter in expanded form, simplified form, and in words.

Example 2

- Find the perimeter of each of the two polygons. The lengths of each side are in cm.

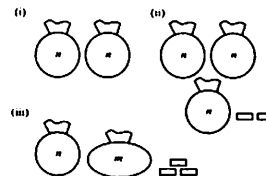


Example 3

- Represent each of the following expressions using a perimeter model
 - $2a + b$
 - $2a + 3$

Bags with coins model

- You have bags containing unknown numbers of coins and some single coins(not in the bags). How many coins are there altogether in each of the following cases?



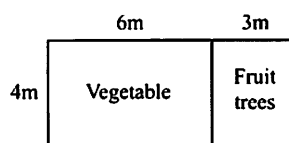
Expansion of the product of a monomial and a binomial

The area model for expansion

The area model can be used to model expansions. The expansion concept is developed from arithmetic, moving to use of one variable, and then to more variables.

Example 1

Students will be asked to consider the following problem. The school garden is partly used for the growing of vegetables and partly for the growing of fruit trees. Calculate the total area of the garden.

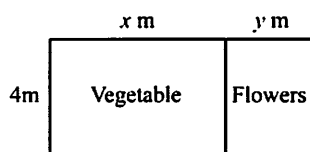


Solution 1: $4 \times (6 + 3) = 4 \times 9 = 36 \text{ m}^2$

Solution 2: $4 \times 6 + 4 \times 3 = 24 + 12 = 36 \text{ m}^2$

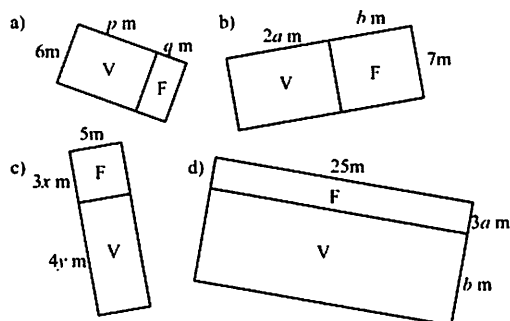
On the discussion board discuss:

- What property allows us to say: $4 \times (6 + 3) = 4 \times 6 + 4 \times 3$
- Explain how each of the solutions are found.
- Rewrite each solution using the following diagram:



Assignment

- Calculate the area of each of the following gardens in two ways:



- Draw vegetable/flower gardens to illustrate these calculations
 - $5 \times (x + 2y) = 5 \times x + 10 \times y$ or $5(x + 2y) = 5x + 10y$
 - $6 \times (6x + 4y) = 36 \times x + 24 \times y$ or $6(6x + 4y) = 36x + 24y$

For c – g expand the product two ways:

- c. $a(x + y) =$
- d. $a(2b + c) =$
- e. $4(2x + y + 3) =$
- f. $2a(3b + c + 5) =$
- g. $2a(a + 3b + 6) =$

Multiplication table algorithm for expansion

The area model is a concrete model. A step towards a more abstract approach is using the multiplication table. This is close to the area model, the difference being that two of the sides of the rectangle are deleted. Below are examples in a multiplication table algorithm format.

$$\begin{array}{c|c|c} \times & 6 & 3 \\ \hline 4 & 24 & 12 \\ \hline \end{array}$$

$4(6 + 3) = 24 + 12$

$$\begin{array}{c|c|c} \times & x & 3 \\ \hline 4 & 4x & 12 \\ \hline \end{array}$$

$4(x + 3) = 4x + 12$

$$\begin{array}{c|c|c} \times & a & b \\ \hline 5 & 5a & 5b \\ \hline \end{array}$$

$5(a + b) = 5a + 5b$

$$\begin{array}{c|c|c} \times & x & y \\ \hline a & ax & ay \\ \hline \end{array}$$

$a(x + y) = ax + ay$

$$\begin{array}{c|c|c} \times & a & -b \\ \hline 5 & 5a & -5b \\ \hline \end{array}$$

$5(a - b) = 5a - 5b$

$$\begin{array}{c|c|c} \times & x & -y \\ \hline a & ax & -ay \\ \hline \end{array}$$

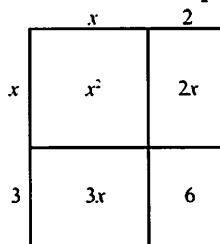
$a(x - y) = ax - ay$

On the discussion board:

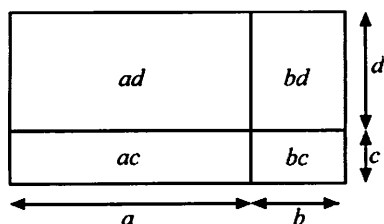
Compare the area model with the multiplication algorithm pointing out advantages and disadvantages of each model. Which would you use in the classroom? Justify.

Expansion of the product of two binomials

Find the area of the following rectangle. Write as an expansion.



Given a rectangle divided into four parts, find two expressions for the total area:



Using Algebra Tiles (from a larger unit that also include discussion of integers)

Students will be directed to an interactive site that will allow them to work with virtual manipulatives. Students will also be asked to make their own with colored card stock and a template provided.

Discussion board activity:

Students will be asked to compare algebra tiles with the area model. What are the similarities?
What are the differences?



Algebra Tiles

- Algebra tiles can be used to model operations involving integers.
- Let the small yellow square represent $+1$ and the small red square (the flip-side) represent -1 .



- The yellow and red squares are additive inverses of each other.

1



Algebra Tiles

- Algebra tiles can be used to model operations involving variables.
- Let the green rectangle represent $+1x$ or x and the red rectangle (the flip-side) represent $-1x$ or $-x$.



- The green and red rods are additive inverses of each other.

2



Zero Pairs

- Called zero pairs because they are additive inverses of each other.
- When put together, they model zero.
- Don't use "cancel out" for zeroes use zero pairs or add up to zero



3



Distributive Property

- Use the same concept that was applied with multiplication of integers, think of the first factor as the counter.

- The same rules apply.

$$3(x + 2)$$

- Three is the counter, so we need three rows of $(x + 2)$

4



Distributive Property

$$3(x + 2) = 3 \cdot x + 3 \cdot 2 = 3x + 6$$



- o Three Groups of x to get three x's
- o Three groups of 2 to get 6

5



Modeling Polynomials

- o Algebra tiles can be used to model expressions.
- o Model the simplification of expressions.
- o Add, subtract, multiply, divide, or factor polynomials.

6



Modeling Polynomials

$2x^2$



$4x$



$3 \text{ or } +3$



7



More Polynomials

- o Represent each of the given expressions with algebra tiles.
- o Draw a pictorial diagram of the process.
- o Write the symbolic expression.

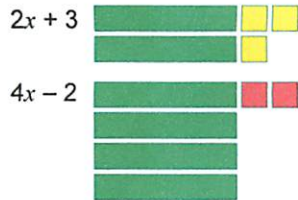
$x + 4$



8



More Polynomials



9



More Polynomials

- Use algebra tiles to simplify each of the given expressions. Combine like terms. Look for zero pairs. Draw a diagram to represent the process.
- Write the symbolic expression that represents each step.
 $2x + 4 + x + 2$

10



More Polynomials

$$2x + 4 + x + 1 = 3x + 5$$



Combine like terms to get
three x 's and five positives

11



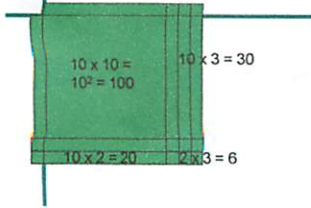
Multiplication

- Multiplication using "base ten blocks."
 $(12)(13)$
- Think of it as $(10+2)(10+3)$
- Multiplication using the array method allows students to see all four sub-products.

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Multiplication using "Area Model"

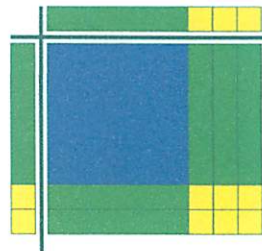
$$(12)(13) = (10+2)(10+3) = 100 + 30 + 20 + 6 = 156$$



13

Multiplying Polynomials

$$(x+2)(x+3)$$



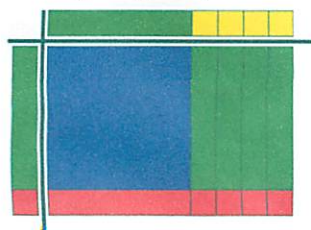
Fill in each section of the area model

Combine like terms

$$x^2 + 2x + 3x + 6 = x^2 + 5x + 6$$

Multiplying Polynomials

$$(x-1)(x+4)$$



Fill in each section of the area model

Make Zeroes or combine like terms and simplify

$$x^2 + 4x - 1x - 4 = x^2 + 3x - 4$$

15

Conclusion

- o Algebra tiles can be made using the Ellison (die-cut) machine.
- o On-line reproducible can be found by doing a search for algebra tiles.
- o Virtual Algebra Tiles at HRW
http://my.hrw.com/math06_07/nsmedia/tools/Algebra_Tiles/Algebra_Tiles.html
 National Library of Virtual Manipulatives
http://nlvm.usu.edu/en/nav/topic_t_2.html

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