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Submission Date: _____
Action-Date: App 12/16/97

CURRICULUM PROPOSAL COVER SHEET
University-Wide Undergraduate Curriculum Committee

Senate app. 2/3/98

I. CONTACT

Contact Person Gerald Buriok Phone 2608

Department Mathematics

II. PROPOSAL TYPE (Check All Appropriate Lines)

X .COURSE MA 271 Intr to Algbrc Struc
Suggested 20 character title

____ New Course* _____
Course Number and Full Title

X Course Revision MA 271 Introduction to Algebraic Structures
Course Number and Full Title

____ Liberal Studies Approval ÷ for new or existing course _____
Course Number and Full Title

____ Course Deletion _____
Course Number and Full Title

X Number and/or Title Change MA 271 Introduction to Algebraic Structures
Old Number and/or Full Old Title

MA 271 Introduction to Mathematical Proofs I
New Number and/or Full New Title

____ Course or Catalog Description Change _____
Course Number and Full Title

____ PROGRAM: ____ Major ____ Minor ____ Track

____ New Program* _____
Program Name

____ Program Revision* _____
Program Name

____ Program Deletion* _____
Program Name

____ Title Change _____
Old Program Name

_____
New Program Name

III. Approvals (signatures and date)

Department Curriculum Committee 9/14/97

Department Chair 9/14/97

College Curriculum Committee 10/15/97

College Dean 10/15/97

÷Director of Liberal Studies (where applicable)

*Provost (where applicable)

Part II. Description of the Curriculum Change

1. New syllabus of record - attached.

2. A summary of the proposed revisions.

MA 271 Algebraic Structures, a one semester course emphasizing proofs in mathematics, will be expanded to form a two semester sequence MA271/272 Introduction to Mathematical Proofs I and II. The content of MA 271 will be changed in that some current topics will be moved to MA 272 and new topics dealing with mathematical proofs will replace them. MA 123 Calculus I for Physics, Chemistry, and Mathematics has been added as a prerequisite.

3. Justification/rationale for the revision.

The faculty of the Mathematics Deparment have agreed to alter both the calculus requirement and the algebraic structures requirement of students in the Mathematics, Applied Mathematics, and Secondary Mathematics Education programs. The calculus change involves deleting MA127/128/227, and requiring instead MA 123/124. The difference between these sequences has been the amount of theory covered versus applications, with a significant theoretical component in the former. The sacrifice of theory in going from the three semester to the two semester sequence will be made up by modifying and expanding the content of MA 271 Algebraic Structures into a two semester sequence, Introduction to Mathematical Proofs I and II. We believe it will be possible to incorporate many of the proofs that would be discussed in a calculus course into this two semester proofs course. MA 123 has been added as a prerequisite to ensure that students have sufficient background in calculus to deal with the topics in the MA 271/272 sequence.

4. Old syllabus of record - attached.

5. Liberal Studies course approval form.

I Catalog Description

MA 271 Introduction to Mathematical Proof I — 3 credits, 3 lecture hours (3c-0l-3sh)

Prerequisites: MA 123 Calculus I; MA 171 Introduction to Linear Algebra

Gives student basic ideas necessary to prove results in mathematics. Includes but is not limited to logic of mathematics, basic methods of proof, algebra of sets, equivalence relations and partitions of sets, functions, and mathematical induction.

II Course Objectives

1. Students study the basics of logic (including use of quantifiers), set theory, relations and functions, and the use of induction.

2. Students will be expected to explore several mathematical topics in order to understand the relation between exploration, discovery, and proof.

3. Students will develop confidence in their ability to read and write formal mathematical proofs.

a. Students will learn to recognize and learn to write various types of mathematical argument: direct, conditional, contrapositive, indirect, mathematical induction

b. Students will learn to write proofs using proof structure: learn to say what they will prove and how, learn to supply a string of logical deductions with justification, learn to summarize what was proved.

c. Students will learn to write proofs using scratch work and develop an initial outline of the proof.

d. Students will learn to polish proofs.

e. Students will learn to read and find flaws in proofs, and to recognize gaps in reasoning in proofs.

III Course Outline

Possible writing assignments are included.

1. Exploration and Proof

1.1. Exploration (1 hour)

General Techniques of Exploration

1.2 From Exploration to Proof (1 hour)

Making Decisions, Why Prove?, Generalization

Writing: timed free-write on why mathematicians do proofs; paragraph on what students learned from reading section in the text.

1.3 Proof: An Introduction (3 hours)

Conditional Sentences, Variables, Open Sentences, Truth Sets, Counterexamples, Definitions, Finding Relationships

2. Exploration and Proofs about Sets and the Integers

2.1 Exploring Sets (2 hours)

Power Set, Cartesian Product, Set Notation, Algebra of Sets, Basics of Cardinality

Writing: have students interchange class notes at end of period and review note taking

2.2 Proofs about Sets (2 hours)

Sets and Subsets, Equality of Sets, Cartesian Products, Cases

Writing: writing set proofs in paragraph form; exercises on finding flaws in proofs

2.3 Exploring the Integers (1 hour)

Multiples and Divisors, Linear Diophantine Equations

2.4 Proofs about Integers (2 hours)

Divisors, Proof by Contradiction, Common Divisors, Common Multiples

Writing: exercise on writing proof in one column in symbols and second column in words

3. The Language and Logic of Mathematics

3.1 Quantifiers, Implication, and Negation (4 hours)

Universal and Existential Quantifiers, Free and Bound Variables, Universal Quantifiers and Implication, Definitions with Quantifiers, Negating Quantified Statements,

Writing: paragraph explaining how to negate quantified statements; timed free-write on how student views mistakes

3.2 Working with Quantifiers (3 hours)

Upper Bounds and Sets Bounded Above, Functions (Domain, Range, 1-1, Onto, Composition)

4. Basic Methods of Proof and Exploration

4.1 Exploration and Proof with Quantifiers (3 hours)

Universal Statement, Existential Statement, Existence Proofs, Quantifiers in the Hypothesis ("for all", "there exists")

Writing: description of types of proofs students have seen in calculus

4.2 What to Prove Instead (4 hours)

Proof by Contradiction, Proof by Contrapositive, Alternative Language for Conditionals, Converse, If and Only If, And, Or, Not, What to Assume?, What to Prove?

5. Induction and the Integers

5.1 The Principle of Induction (3 hours)

Inductive Sets, Axioms For Induction, Proofs by Induction, Generalized Principle of Induction, Definition, by Induction

Writing: induction proofs

6. Relations and Functions

6.1 Arbitrary Relations and Functions (3 hours)

Relations, Functions, Inverse Functions and Composition, General Properties of Relations (symmetry, antisymmetry, reflexivity, irreflexivity, transitivity)

Writing: set proofs involving Cartesian products; assignment on Kleiner's article surveying evolution of function concept; writing proofs that functions are one-to-one, onto, not one-to-one, not onto; paragraph on, how to find inverse of a function; paragraph on how to find domain and range of a composition; proof that composition of one-to-one functions is one-to-one; proof that composition of onto functions is onto; paragraph describing types of relations with examples; exercise on finding flaws in proofs; writing proofs involving relations and supplying examples to disprove statements about relations

6.2 Partitions and Equivalence Relations (4 hours)

Partitions, Making a Relation a Partition, Equivalence Relations, Equivalence Relations vs. Partitions, Congruence modulo n

Writing: paragraph describing relation between equivalence relation and partition with student's own example; proofs that relation is equivalence relation

IV Evaluation Methods:

A guideline for determining the grade for this course follows:

55% Examinations including tests, quizzes, and final exam. All exams will focus on writing mathematical proofs and describing mathematical concepts.

35% Assignments to hand in. This is primarily the writing of proofs but also includes "proofs to grade" and other graded writing assignments.

10% Class participation and non-graded writing assignments. This will be based on the number of non-graded writing assignments completed, problems or proofs presented in class at the board, and participation in other classroom activity.

V Required Text Book

Fendel, Daneil & Resek, Diane, *Foundations of Higher Mathematics: Exploration and Proof*, 1st ed., Addison Wesley, 1990.

VI Special Resource Requirements

None

VII Bibliography

Polya, How to Solve it,

Smith, et. al., A Tranisition to Advanced Mathematics, Brooks/Cole

Notes:

This outline covers 36 hours, leaving 6 hours for tests, review, etc. This course is a prerequisite for the 300 level mathematics courses MA 353 (Number Theory), MA 355 (Geometry), and MA 371 (Linear Algebra). The prerequisite includes a grade of "C" or better. Required of all mathematics majors.

The "Guide for Written Homework" which appeared in the article "Learning Mathematics Through Writing: Some Guidelines" by J. J. Price (College Mathematics Journal, November 1989) or similar substitute will be distributed at the beginning of the course.

STATEMENT CONCERNING DEPARTMENT RESPONSIBILITY

The Mathematics/Applied Mathematics Committee will have general responsibility to assure that MA 271 Introduction to Mathematical Proof I is taught in a manner consistent with the writing-intensive course outline and description.

The Mathematics Department Chairperson, in consultation with the Mathematics/Applied Mathematics Committee, will be responsible for assigning the course to Mathematics faculty members who are aware of the nature of writing intensive courses and committed to integrating writing into the course content.

In general the course will be assigned only to Mathematics faculty who have completed an approved writing workshop. However, in any case that the course is assigned to a faculty member who has not completed a writing workshop, the Chairperson will be responsible for assuring that the faculty member is committed to writing as an element in learning mathematics and is aware of various approaches to integrate writing into mathematics courses. The latter will be achieved by expecting the faculty member to have read at least the articles 1, 2, 3, 4, 6, 7, 11 from the book Writing to Learn Mathematics and Science edited by Paul Connolly and Teresa Vilardi.

COURSE SYLLABUS

Course Number: MA 271

Course Title: Introduction to Algebraic Structures

Credits: 3 semester hours

Prerequisites: MA 171

Textbook: After Calculus: Algebra by Foulis and Munem

Revised: 1/90

Catalog Description:

Gives student basic ideas of contemporary mathematics. Includes mathematical logic, algebra of sets, equivalence relations and partitions of sets, functions, and fundamentals of group theory. Methods of proof in abstract mathematics are emphasized.

Objectives:

1. Students will study the basics of logic (including the use of quantifiers), set theory, relations and functions, and the definition and simple properties of a group.

2. Students will develop confidence in their ability to read and to write formal mathematical proofs.

a. Students will learn to recognize and learn to write various types of mathematical argument: direct, conditional, contrapositive, indirect, math induction.

b. Students will learn to write proofs using proof structure: learn to say what they will prove and how, learn to supply a string of logical deductions with justification, learn to summarize what was proved.

c. Students will learn to write proofs using scratch work and developing an initial outline of the proof.

d. Students will learn to polish proofs.

e. Students will learn to read and find flaws in proofs, and to recognize gaps in reasoning in proofs.

Course outline with approximate time schedule and brief indication of writing assignments:

I. Introduction to logic and proofs

A. Elements of propositional calculus including truth tables and rules of inference (3 periods)

Writing: timed free-write on why mathematicians do proofs; paragraph on what students learned from reading this section in text

B. Quantifiers and negations of quantified statements (2 periods)
Writing: paragraph explaining how to negate quantified statements; timed free-write on how student views mistakes

C. Two column logic proofs and introduction to types of proof (direct, conditional, contrapositive, indirect, by cases, existence) (4 periods)
Writing: Description of types of proofs students have seen in Calculus; writing of two column logic proofs; paragraph on meaning of definition of limit; exercise on writing proof in one column in symbols and second column in words

D. Introduction to sets (1 period)
Writing: have students interchange class notes at end of period and review note taking

E. The algebra of sets including 2 column set proofs and the introduction to paragraph proofs of set theorems (5 periods)
Writing: writing 2 column set proof of theorem, writing 2 column and paragraph proof for same theorem, writing paragraph proofs; exercises on finding flaws in proofs

F. Generalized unions and intersections using indexed families (2 periods)
Writing: Paragraph describing use of indexed family to write arbitrary union; exercise using double entry format on historical notes at the end of the chapter

II. Relations and Functions.

A. Ordered pairs and the cartesian product (1 period)
Writing: set proofs involving cartesian products

B. Binary relations (reflexive, transitive, symmetric, antisymmetric) and partial orders (4 periods)
Writing: paragraph describing types of relations with examples; exercise on finding flaws in proofs; writing proofs involving relations and supplying examples to disprove statements about relations

C. Partitions and equivalence relations (3 periods) -
Writing: paragraph describing relation between equivalence relation and partition with student's own example; proofs that relation is equivalence relation

D. Functions including injections, surjections, bijections and inverse functions(3 periods)
Writing: assignment on Kleiner's article surveying evolution of

function concept; writing proofs that functions are injective, surjective; paragraph on how to find inverse of a function.

E. Composition of functions (1 period)
Writing: paragraph on how to find domain and range of a composition; proof that composition of injections is an injection, proof that composition of surjections is surjection

III. Algebraic systems

A. Binary operations, simple algebraic systems and isomorphism (3 periods)
Writing: paragraph on meaning of isomorphism and why they should preserve properties; proofs that isomorphisms preserve specific system properties

B. Group properties (3 periods)
Writing: proofs that an algebraic system is or is not a group; proofs that an algebraic system is or is not isomorphic to a group

IV. Mathematical Induction (2 periods)
Writing: induction proofs

Note: The above schedule allows 4 periods for tests, and 1 period for final review.

Evaluation Methods:

The grade for this course will be determined approximately as follows:

55% Examinations including 3 tests, quizzes, and final exam. All exams will focus on writing mathematical proofs and describing mathematical concepts.

35% Assignments to hand in. This is primarily the writing of proofs but also includes "proofs to grade", and other graded writing assignments.

10% Class participation and non-graded writing assignments. This will be based on the number of non-graded writing assignments completed, problems or proofs presented in class at the board, and participation in other classroom activity.

Note: This course is a prerequisite for many upper level mathematics courses. This prerequisite includes a grade of "C" of better.

WRITING SUMMARY FOR MA 271
INTRODUCTION TO ALGEBRAIC STRUCTURES

Three types of writing will occur in this course.

1. WRITING MATHEMATICAL PROOFS.

The primary focus of the course is to develop the ability of students to write clear mathematical proofs in the format used by most mathematicians. The style is very terse with short sentences. Precision in the use of mathematical terms is extremely important. (Readers should be reminded that the rules of grammar, spelling, and punctuation apply to mathematics. "a + b = 15." is a sentence while "a + b" is not a sentence.) At least 30% of the grade will be based on hand-in assignments that involve writing or evaluating proofs. A significant number of these assignments will require revision. Revisions will be handled in two different ways. In some cases students will be required to hand in two drafts simultaneously, a rough draft as well as a more polished proof. In some cases students will turn in a completed proof, which will be returned with comments before further revision.

2. PROSE WRITING TO CLARIFY THINKING, BUILD INTUITION AND EVALUATE INSTRUCTION.

After reading the text and hearing class discussion of a mathematical concept, students will be asked to summarize that concept in their own words without using mathematical symbolism. The concepts for which this technique will be used include: negation of quantifiers, limits, arbitrary unions and intersections of sets, types of mathematical argument, relations, partitions, bijections. This writing is designed to force students to concentrate on understanding the concept and to clarify their thinking about mathematical terms. Questions of the above kind will be collected as homework and will also appear on examinations.

Writing that will be collected and reviewed but not graded will include the following: timed free-write on why mathematicians do proofs, timed free-write on attitude toward mistakes, class notes, written questions on what students did not understand about a certain reading assignment in text, paragraph on the biggest problem with assignment just turned in, short essays written during the last 5 minutes of class explaining concept discussed in class or discussing question uppermost in students mind.

3. WRITING TO ENHANCE READING.

Students will write one short paper (usually a reaction paper) on the readings in the Mathematics Department Reading Program. Students will write a second short paper to discuss the article by Israel Kleiner:"Evolution of the Function Concept: A Brief Survey". For this

paper students will be asked to use a double entry format with one column for summary and one column for their own reaction/response. Students will write a third short paper based on an article from a mathematics journal. Students will be asked to discuss the techniques used in the paper and to describe a short mathematical proof using a double entry format with one column for mathematical symbols and one column for their own words describing the meaning of the symbols.

From: GROVE::JBURIOK
To: CCULLUM
CC: JBURIOK
Subj: Screening Committee LS Questions

Charles: I sent Darlene Richardson a message about the items your screening committee said needed LS approval. I will forward that message to you. Below is Darlene's reply. Is this enough of a response to your committee's concerns involving MA 271, MA 350, MA 123, and MA 124? If not, please tell me what I need to do.

Jerry Buriok

From: GROVE::DRCHRDSN "Darlene Richardson, Liberal Studies" 14-NOV-1997 11:32:58.92
To: GROVE::JBURIOK
CC: DRCHRDSN, CCULLUM
Subj: RE: LS questions - Screening Committee

Hi, Jerry. The designation of a course as writing-intensive is not part of the approval process of the course (reference to MA 271 and MA 350). The reasons for this is that a course may be taught writing-intensive one semester but not another; another reason is that one section of the course may be writing-intensive, but another section may be non-writing-intensive even in the same semester. You have notified the LS office that the objectives and writing assignments in MA 271 and MA 350 will remain the same, so that the Type II writing approval continues. Because the content and objectives of MA 123 and 124 are essentially the same (just a name change), simply send a memo to that effect to the LS office. Darlene

MAIL>