is difficult at best, and, even with a positive outlook, some language skill and extended-family support in place, parenting in Pittsburgh is difficult for LEP parents. The parents who participated in this survey told us how they manage, but they speak for themselves, not for all ESL students. Let's keep asking the students what they want!

Using Tic-Tac-Toe Math: A Case Study

Patricia Y. Pisaneschi

The Story of the Question

For the past three academic years I have worked part-time as a mathematics instructor in an adult literacy program sponsored by Luzerne County Community College. Students enrolled in the program meet for two, 2-hour sessions each week, and those students at the GED or upper ABE level focus on math during one of these weekly sessions.

Although my bachelor's degree was in mathematics, I had not begun to teach math in the classroom until 1995, when I began serving LCCC as an adjunct instructor in developmental mathematics courses. In the spring of 1997 I went to a workshop presented by Dr. Richard Cooper and was fascinated by his alternative methods of presenting math. During the summer several teachers left the adult literacy program and I was assigned, somewhat by default, to an ABE group of students who were working at a low level in both reading and math. Most of these students added reasonably well and subtracted with just a little difficulty on large numbers. Multiplication and division, however, were quite difficult for all of them.

We worked on simple word problems, mostly addition and subtraction, to help build both reading and math skills. Another teacher was assigned to share the class, and the group got so large that no one was

---

Patricia Pisaneschi is an Instructor of Adult Basic Education/GED with the Luzerne County Community College, Mountaintop, PA.

getting the individual attention that is even more critical with ABE students than with the students working on a higher level. The class was split, and the upper level ABE group was still assigned to both of us, but we (and the students) agreed that two sessions of reading would be more appropriate for the lower level group. However, I'd gotten to know these students and missed working with them, and I was especially interested in one bright young woman (let's call her J) who had joined the class. J had already received a high school diploma from participating in special education classes, but she entered our program as a non-reader who was unable to do multiplication or division without the aid of a calculator. However, she was quick to identify the process necessary to solve simple word problems if the problems were read aloud to her. Might she be a good candidate for Dr. Cooper's methods?

After discussion with my co-teacher and agreement from our program director, who was understandably concerned that all program clients should be treated equally with regard to the available amount of class time, we decided that since I had a 1½-hour break between classes held at the same location, I would volunteer part of my break time to work with students from the newly split class on an individual or small-group basis once each week while my co-teacher worked with the remaining students in a more individualized setting. Thus, the students would be able to continue with math, but they still would work for more than one session on their reading.

I wanted to meet J alone because I had discovered that, when I included her in the small group, she answered the problems read aloud by the other group members before they had time to figure out how to set them up. I knew that she was just learning to identify letters and their sounds, and she explained that she sometimes reversed numbers and had trouble remembering multiplication facts. Could she be a classic learning disabled student? I was not, and still am not, concerned about what labels may be appropriate for her learning difficulties, but I was very excited about finding out what techniques could help her learn.

Now seemed to be a good time to delve into the Tic-Tac-Toe math book that I'd purchased and that explained Dr. Cooper's techniques for teaching multiplication. For readers not familiar with this method, it involves setting up a tic-tac-toe grid for each number using the following pattern:

1	4	7
2	5	8
3	6	9

The grid for each number is set up in the same fashion:

2	8	14	3	12	21
4	10	16	6	15	24
6	12	18	9	18	27

Note that the opposite numbers always add up to 10 times the original number: $2 + 18, 4 + 16; 9 + 21, 12 + 18$. We can see how this "magic" works if we write the grid in algebraic format:

1N	4N	7N
2N	5N	8N
3N	6N	9N

Thus, 2N plus 8N is always equal to 10N, 3N plus 7N is always equal to 10N, etc. The central number always ends in 5 or 0 (it is always 5N) and is, of course, always equal to one half of 10N. Tables for the odd and even numbers create distinctive patterns with the last digit of each table value, and learning these patterns makes it possible for students to construct the multiplication tables, rather than memorize them. Thus, the student who is unable to memorize basic multiplication facts is now empowered to construct them without the aid of written tables or a calculator.

I had learned that J was interested in art, and, after seeing one of her drawings, I suspected that the patterned approach involved in Tic-Tac-Toe (TTT) tables might appeal to her. The only problem was that I had difficulty understanding Dr. Cooper's written explanation! I explained to J that although I was only learning the technique myself, I thought that it just might work for her. So we sat down together, and I began explaining the way Cooper sets up multiplication tables. Lo and behold, I began to understand what he was doing! And even more important, J began to understand, too.

As J began to make progress, I found myself wondering whether other students could also make good use of the TTT technique. Some of the slower students indicated that they had tried in the past to learn the multiplication facts, but without success. Most of them were still counting to figure out the correct (sometimes incorrect) products. So my question expanded to include use of the TTT method with more than one student.

How does TTT work for students? Is this method useful for teaching my students to learn and use multiplication tables? Can my "classic" learning disabled student make progress by using it? Also, can my slow learners understand it? Would it help somewhat more advanced students with the tables they have trouble with, usually tables for numbers bigger

than 5? What points will I have to keep in mind as I try to teach this method to J and other selected students?

Data Collection Methods

My primary data collection method was a journal of entries made after each session with J or with other students. In this journal I recorded notes about what we had done, what kind of progress had been made, what questions were asked, and what kinds of personal comments were shared. In addition, I asked informal interview questions based on curiosity about their backgrounds, previous experiences with math, and ways in which they use math at present. Notes on their responses were also entered in my journal.

Findings

Most of the time during this study was spent with J, not only because of my own enthusiasm but because, if I did not go to call her into my classroom, she came looking for me. We began by examining the patterns used to set up the tables from 1 to 9, and she quickly learned to use them for multiplication and for division, which she told me she'd never actually been able to do. We moved rapidly on to constructing tables for 2-digit multipliers. As I showed her how to write the tables for each digit of the multiplier side by side, she noticed that, as I copied the table for the one's digit, I was writing the one's digit for each product in large print and any ten's digit in small print (essentially, to "carry" the number along, making it easy to see and add to the value from the table for the ten's digit in the next step). She asked why I was doing that and, before I explained went on to say, "Oh! I think I know!" Sure enough, as I started to write the table for the ten's digit on the same grid, she answered her own question as she told me to add each table value for the ten's digit to any small number already noted in the appropriate box before writing the value in the new table for the 2-digit multiplier.

Extensive drill on similar problems did not seem necessary for this student. It became very clear early on that she was willing to ask questions and would tell me if something was not clear to her. On her first set of 16 division problems she had only 2 errors, both involving zeroes in division answers, and pointed out (correctly) that I hadn't shown her that. My notes are filled with comments like these: "What a JOY to work with her!" "J is amazing!" "What a quick mind!" It's sad that no one ever

showed her some alternate methods when she was attending public school. Of course, as I told her once, she was probably a lot harder to deal with as a teen. She had no trouble agreeing with that perception and reported that in junior high she had stolen the math teacher's book with answer keys—not exactly an endearing behavior.

I discovered that J did not like having me correct her errors in my famous green pen; she preferred to erase them and do the corrections herself. How wonderful to find a student who WANTED to make corrections! Very quickly J mastered the construction of tables for multiple digits and demonstrated her ability to use them in multiplication and division problems. We moved into an explanation of how to work with decimals, and one day J came in very concerned because she simply could not get the right answer for the problem 4863.76 divided by 9.2, a deliberately lengthy problem I had made up for her. When I looked closely at it, it all seemed correct; then she informed me that her calculator disagreed about the tenth decimal place. Of course, the calculator had stopped by rounding up, while she was still continuing the problem! Such are the difficulties with problems made up off the top of one's head.

After mastering 3-digit tables, J decided she'd like to try fractions, which she said she had never understood anything about. We continued to use the tables as she learned to reduce fractions, and she continued to use her thinking skills to master fractions quickly. She discovered that 7 was a probable divisor for 56/63 by finding the difference between the two numbers. She even made up some of her own problems, like 78/68—not a problem I'd likely assign to anyone.

We next covered multiplication and division of fractions, throwing in some mixed numbers as we went along, and then we went to addition and subtraction. Up to now I had been writing problems for J in large print. At this point I thought it would be useful for her to see that she could do the kinds of fraction problems that other students are expected to master in a pre-GED book. I thought I could enlarge relevant pages on the copier, but that still did not leave enough room for her to work the problems on the page. She suggested that she could now start to copy the problems herself as long as they were somewhat larger than the print in the book. This also meant she could write the addition and subtraction problems vertically instead of horizontally—another new step! She was able to copy assignments of about 20 to 30 problems correctly and solve the majority of them without difficulty (although she did disclose that a number of sheets of paper had been crumpled and torn up while she was completing her assignment at home).

All of J's accomplishments described here took place in a period of about 8 weeks, with our meetings lasting from 15 to 60 minutes, and occurring usually once, and sometimes (often at J's request) twice, each week. Clearly, the TTT tables have unlocked the door for this student to use her considerable intelligence to solve mathematical problems.

The results of using the TTT tables with other students are not so clear cut. Student X is an older man and a slow learner; he is reading at a low elementary level and has difficulty retaining what he is studying. However, X has good basic skills in math through multiplication, but he has been using a traditional table and still has some problems multiplying larger numbers. He also has some trouble remembering how to set up the TTT tables, but he is becoming familiar with their use in solving problems.

Student Y, also an older man, is working at an upper-ABE level and is making good progress in reading. He works slowly and deliberately and has mastered whole number multiplication and division with large numbers; however, he is also dependent on tables and is currently using the TTT tables to solve problems.

Student Z is a young mother having some difficulties studying at the GED level, especially in math. Like X and Y, she has trouble remembering multiplication facts for numbers larger than 5. She seemed to grasp quickly the procedure for setting up TTT tables and checking them for accuracy; she is just starting to use them to see if they are effective for her in solving problems.

The goal for students X, Y, and Z is that they learn how to construct the TTT tables if, in fact, they find them easy to use. Some early indications indicate that this will be so, but more work is needed to be sure they are able to construct the tables accurately.

Implications

It is clear from my work with J that the TTT tables can work with some students. My work with the other students indicates possibilities, but the effort demonstrates that slower learners are also likely to be slower at understanding and learning to construct the TTT tables. This is hardly a surprising finding, but what I hope to discover soon is whether a modified approach, such as constructing the tables only for 6, 7, 8, and 9, is actually an effective tool for learners having difficulties. As I explain to students, TTT is not a shortcut; rather, it represents a detour that enables us to bypass a poor memory. Is it appropriate to present TTT to students in

partial form? Only further observations will provide more information on this aspect of TTT.

However, the most exciting part of this project has been the privilege of seeing J become an empowered learner through the use of TTT. This student entered our program because she was tired of feeling stupid. She once went shopping in the grocery store and bought dog biscuits instead of crackers because she couldn't read the labels. She has mistakenly used Brill Creme instead of toothpaste to brush her teeth and taken Efferdent instead of Alka-Seltzer because the packages looked the same.

Yet J is one of the least stupid students I've ever met! Although she has difficulty remembering the numbers to dial the phone, she has been able to memorize the pattern of frequently called numbers. She has good logical reasoning ability, as evidenced by her ability to understand math problems and other material presented orally. Her grasp of concepts, analogies, and other explanations is quick and precise. Her art work exhibits fine detail and visual perception.

J is making steady progress in reading, writing, and computer skills along with her phenomenal development in math. Despite having been largely ignored academically in public school, despite being given mainly negative attention both at home and at school, and despite emotional concerns as well as family difficulties, she has not given up. Even among adult students, who are generally noted for their motivation and persistence, J stands out.

J always likes to know why something is true, so recently I explained to her the logic of the TTT tables, using the 1N, 2N, . . . 10N explanation. She quickly grasped the idea and then made the following request: "Will you show me more about working with those letters?"

Algebra, here we come!