LSC Use Only
Number:
Submission Date:
Action-Date:

UWUCC USE Only
Number: 97-35g
Submission Date:
Action-Date: App. 12/16/97
Senate app. 2/3/98

CURRICULUM PROPOSAL COVER SHEET
University-Wide Undergraduate Curriculum Committee

I. CONTACT

Contact Person Gerald Buriok Phone 2608

Department Mathematics

II. PROPOSAL TYPE (Check All Appropriate Lines)

x COURSE ~~MA 272 Intro Math Proofs II~~
Suggested 20 character title

X New Course* MA 272 Introduction to Mathematical Proofs II
Course Number and Full Title

___ Course Revision ___
Course Number and Full Title

___ Liberal Studies Approval + for new or existing course ___
Course Number and Full Title

___ Course Deletion ___
Course Number and Full Title

___ Number and/or Title Change ___
Old Number and/or Full Old Title

___
New Number and/or Full New Title

___ Course or Catalog Description Change ___
Course Number and Full Title

___ PROGRAM: ___ Major ___ Minor ___ Track

___ New Program* ___
Program Name

___ Program Revision* ___
Program Name

___ Program Deletion* ___
Program Name

___ Title Change ___
Old Program Name

___
New Program Name

III. Approvals (signatures and date)

[signature] 9/14/97
Department Curriculum Committee

[signature] 9/14/97
Department Chair

[signature] 10/15/97
College Curriculum Committee

John D. Eck 10/15/97
College Dean

___
+Director of Liberal Studies (where applicable)

___
*Provost (where applicable)

Course Analysis Questionnaire

Section A: Details of the Course

A1 How does this course fit into the programs of the department? For what students is the course designed? (majors, students in other majors, liberal studies).

The core program for students majoring in Mathematics, Applied Mathematics, and Secondary Mathematics Education currently consists of the 3 four credit calculus courses MA127, MA128, MA227, along with the 2 three credit courses MA171 Introduction to Linear Algebra and MA271 Algebraic Structures. The revised program will replace the calculus sequence with 2 four credit calculus courses, MA123 and MA124, continue MA171, and expand MA271 into a two semester sequence MA271 and MA272. Thus MA272 will be a required course for students in all of the undergraduate programs offered by the Mathematics Department.

A2 Does this course require changes in the content of existing courses or requirements for a program? If catalog descriptions of other courses of department programs must be changed as a result of the adoption of this course, please submit as separate proposals all other changes in courses and/or program requirements.

The request for approval of MA272 Introduction to Mathematical Proof II is accompanied by a request for approval of a revision to the existing course MA271 Algebraic Structures. There will be a name change to MA271 Introduction to Mathematical Proof I, as well as a change in course content. In addition, MA421 Advanced Calculus I, MA427 Introduction to Topology, and MA476 Abstract Algebra I all have "C or higher in MA271" as their current prerequisite. Requests for approval of revisions of prerequisites of these courses to "C or higher in MA272" also accompany the request for approval of MA272.

The core program for students in all three undergraduate programs offered by the Mathematics Department, namely Mathematics, Applied Mathematics, and Secondary Mathematics Education, will be revised so that both MA271 and MA272 are required instead of just MA271, which is the current requirement.

A3 Has this course ever been offered at IUP on a trial basis (e.g. as a special topic)? If so, explain the details of the offering.

No.

A4 Is this course to be a dual-level course? If so, what is the approval status at the graduate level?

This course is not to be a dual-level course.

A5 If this course may be taken for variable credit, what criteria will be used to relate credits to the learning experience of each student? Who will make this determination and by what procedures.

This course may not be taken for variable credit.

A6 Do other higher education institutions currently offer this course? If so, please list examples.

Virtually all Mathematics Departments which teach courses for mathematics majors deal with the concepts of proofs in mathematics. Some institutions, such as the University of Pittsburgh (MA 041) and Slippery Rock U. of PA (MA 131) have a one semester course, while Penn State (MA 311 and MA 312) has a two semester sequence in which students learn to analyze and write proofs.

A7 Is the content, or are the skills, of the proposed course recommended or required by a professional society, accrediting authority, law or other external agency? If so, please provide documentation. Explain why this content or these skills cannot be incorporated into an existing course.

The content of the proposed course is not required by a professional society, accrediting authority, law, or external agency. The content is recommended in the 1991 report of the Mathematical Association of America Undergraduate Programs in Mathematics Programs (CUPM) Guidelines: The Undergraduate Major in the Mathematical Sciences.

In typical undergraduate mathematics programs, the entry course for students is calculus. Although there is some theory involved, much of calculus involves manipulative skills. On the other hand, upper level courses in pure mathematics are theoretical and involve proofs and theorems rather than manipulations. Courses such as MA 271 and MA 272 serve to aid students in the transition from courses involving manipulation to courses involving theory. Generally this transition is difficult for students and separate courses to assist in the transition are necessary.

<u>Section B: Interdisciplinary Implications</u>

B1 Will this course be taught by one instructor or will there be team teaching? If the latter,

explain the teaching plan and its rationale.

This course will be taught by one instructor.

B2 What is the relationship between the content of this course and the content of courses offered by other departments? Summarize your discussions (with other departments)concerning the proposed changes and indicate how any conflicts have been resolved. Please attach relevant memoranda from these departments which clarify their attitudes toward the proposed change(s)?

There is no relationship between this course and the content of courses offered by other departments.

B3 Will seats in this course be made available to students in the School of Continuing Education?

No.

Section C: Implementation

C1 Are faculty resources adequate? If you are not requesting or have not been authorized to hire additional faculty, demonstrate how course will fit into the schedules of current faculty. What will be taught less frequently or in fewer sections to make this possible?

Inclusion of this course in the Mathematics and Applied Mathematics programs is part of an overall revision of these programs. This revision involves replacing the three semester, twelve credit, calculus sequence MA127, MA128, and MA227 with the two semester, eight credit, calculus sequence MA123 and MA 124. It also involves replacing the one semester course MA271 Algebraic Structures with a two semester sequence MA271 and MA272, three credits each. Thus we decreases students' requirements by four credits in calculus and increase requirements by three credits in algebraic structures (mathematical proofs). Inclusion of a one credit senior seminar for majors in Mathematics and Applied Mathematics brings requirements in these programs to the current level, while increasing the number of credits for MA350 History of Mathematics from 2 credits to 3 credits brings requirements for the Secondary Mathematics Education program back to the current level. (The proposed revision to change MA350 from 2 credits to 3 credits accompanies this proposal.) But overall, the faculty workload in terms of credit hours will not increase with approval of the proposed program revisions.

Since there is no change in the number of credits required by students, there is no need for additional faculty. The senior seminar will result in an additional preparation, but this is easily handled by assigning a faculty member to

teach two sections of a four credit calculus courses, one three credit course, and the senior seminar.

C2 What other resources will be needed to teach this course and how adequate are the current resources? If not adequate, what plans exist for achieving adequacy? Reply in terms of the following: Space; Equipment; Laboratory supplies and other consumable goods; travel funds.

No additional resources will be needed to teach this coures. Current resources are adequate.

C3 Are any of the resources for this course funded by a grant? If so, what provisions have been made to continue support for this course once the grant has expired? (Attach letters of support from Dean, Provost, etc.)

None of the resources for this course will be funded by a grant.

C4 How frequently do you expect this course to be offered? Is this course particularly designed for or restricted to certain seasonal semesters?

This course will be offered each semester.

C5 How many sections of this course do you anticipate offering in any single semester?

One section will be offered each semester. The Mathematics, Applied Mathematics, and Secondary Mathematics Education programs are currently at enrollment levels for which one section each semester will satisfy student demand.

C6 How many students do you plan to accommodate in a section of this course? Is this planned number limited by the availability of any resources? Explain.

In order to serve the purposes of the course, enrollment will be limited to thirty students. It is unlikely we will approach this number in the near future.

C7 Does any professional society recommend enrollment limits or parameters for a course of this nature? If they do, please quote from appropriate documents.

There is no evidence of any professional society recommending enrollment limits for a course of this nature. In most mid-level and upper-level mathematics courses, high enrollments are not a problem.

Section D: Miscellaneous

No additional information.

I Catalog Description

MA 272 Introduction to Mathematical Proof II — 3 credits, 3 lecture hours (3c-0l-3sh)

Prerequisites: MA 124 ; MA 271 with a "C" or better

A further study of the basic ideas of contemporary mathematics. Topics include but are not limited to mathematical induction, cardinality of sets, relations, methods of proof in number theory, analysis, and algebra.

II Course Objectives

1. Students apply the basics of logic (including use of quantifiers), set theory, relations and functions, and the use of induction.

2. Students will be expected to explore several mathematical topics in order to understand the relation between exploration, discovery, and proof.

3. Students will continue to develop confidence in their ability to read and write formal mathematical proofs.

   a. Students will learn to recognize and learn to write various types of mathematical argument: direct, conditional, contrapositive, indirect, mathematical induction

   b. Students will learn to write proofs using proof structure: learn to say what they will prove and how, learn to supply a string of logical deductions with justification, learn to summarize what was proved.

   c. Students will learn to write proofs using scratch work and develop an initial outline of the proof.

   d. Students will learn to polish proofs.

   e. Students will learn to read and find flaws in proofs, and to recognize gaps in reasoning in proofs.

4. Students will be introduced to topics from several upper level courses including algebra, topology and advanced calculus.

III Course Outline

Suggested exercises are given.

Induction and the Integers

5.1 (review) and 5.2 The Principle of Induction (6 hours)

Inductive Sets, Axioms For Induction, Proofs by Induction, Generalized Principle of Induction, Definition by Induction, The Second Principle of Induction, Existence of Prime Factorization, Well-Ordering Principle, Division Algorithm, Relationships among the Principles

5.2 Exercises: 1, 3, 5, 6, 9, 12, 13, 16

5.3 Basic Number Theory (2 hours)

Linear Diophantine Equations, GCD, Uniqueness of Prime Factorization, Nonunique Factorization

Exercises:1, 2, 3, 4, 5, 7, 8

Relations and Functions

6.2 (Review) Partitions and Equivalence Relations (1 hour)

Partitions, Making a Relation a Partition, Equivalence Relations, Equivalence Relations vs. Partitions, Congruence modulo n

Further Ideas on Functions and Relations

7.1 Binary Operations (3 hours)

Definition of Operation, Properties of Operations (commutativity, associativity, identity element, inverse element), Arithmetic in Zn, Well-Defined Functions

Exercises: 1, 2, 3, 4, 6, 9, 10, 12, 13, 14, 15

7.2 Order Relations (2 hours)

Include a brief review of 3.2 and 4.1, Subset Relation, Partial Ordering, Total Ordering, Generalizing Upper Bounds, Strict Ordering

Exercises: 1, 2, 3, 7, 8, 9, 10

Cardinality

8.1 Cardinality (3 hours)

Formalizing Cardinality (equivalence relation), A Partial Ordering for Cardinalities, Cantor-Schroeder-Bernstein Theorem (no proof), Infinite Sets (countable, uncountable), Cardinality and Power Sets

Exercises: 1, 2, 3, 4, 5, 6, 7, 8, 9, 10

*8.2 Proof of the Cantor-Schroeder-Bernstein Theorem (2 hours)

Exercises: 1, 2

Limits and Continuity

9.1 Limit of a Sequence-Developing a Definition (2 hours)

Lead the students to a definition, including tentative definitions

Exercises: 1, 2, 3, 4, 5

9.2 Limit of a Sequence-Using the Definition (2 hours)

Examples (algebraic simplification), Sums of Sequences, Uniqueness of Limits, Sequences with No Limits (Limit is infinity is in exercises)

Exercises: 1, 2, 3, 5, 6, 7, 8, 9, 10, 11, 12, 14, 15, 17

9.3 Limits of Functions and Continuity (4 hours)

Limit of a Function, Arithmetic of Limits, Continuity, Intermediate Value Theorem

Exercises: 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 12, 14

*9.4 Uniform Continuity and Cauchy Sequences (3 hours)

Uniform Continuity, Heine-Borel Theorem, Cauchy Sequences

Exercises: 1, 2, 4, 5, 6

Groups

10.1 Groups-Examples and Definitions (4 hours)

Exploration-Developing a Definition, Symmetric group on n Letters, Invertible Elements, Power Sets and Union, Cartesian Product, Dihedral Groups, Exploring Finite Groups

Exercises: 1, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12

10.2 Basic Theorems about Groups (4 hours)

Operation Tables, Cancellation Laws, Subgroups, Cyclic Groups, Order of an Element, Lagrange's Theorem, Cosets

Exercises: 1, 2, 3, 4, 5, 6, 8, 9, 10, 11

10.3 Isomorphism (2 hours)

Comparing Groups, Isomorphism, Isomorphism as an Equivalence Relation, Isomorphism of Cyclic Groups

Exercises: 1, 2, 3, 4, 5, 6, 10, 11, 12

IV Evaluations Methods

A guideline for determining the grade for this course follows:

55% Examinations including tests, quizzes, and final exam. All exams will focus on writing mathematical proofs and describing mathematical concepts.

35% Assignments to hand in. This is primarily the writing of proofs but also includes "proofs to grade" and other graded writing assignments.

10% Class participation and non-graded writing assignments. This will be based on the number of non-graded writing assignments completed, problems or proofs presented in class at the board, and participation in other classroom activity.

V Required Text Book

Fendel, Daneil & Resek, Diane, *Foundations of Higher Mathematics: Exploration and Proof*, 1st ed., Addison Wesley, 1990.

VI Special Resource Requirements

None

VII Bibliography

Polya, How to Solve it,

Smith, et. al., A Tranisition to Advanced Mathematics, Brooks/Cole